THE JOINT LAMINAR CONVECTION OF A BINARY MIXTURE NEAR A VERTICAL SURFACE

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Joint (free and forced) convection near a vertical surface is studied. The one-dimensional heat and mass transfer of a binary mixture is described by a system of differential equations for the boundary layer taking into account diffusion thermal conductivity. The approach to solving the problem varies in accordance with the nature of the basic flow.

NOTATION

x, y – coordinates
g – gravitational acceleration
T – the temperature
ν – kinematic viscosity
β_T, β_M – coefficients of thermal and concentration expansion
a – thermal diffusivity
M – the molecular weight
h – coefficient of mass release
D – the diffusion coefficient
a_T – thermal diffusion constant
R_1 – the gas constant
c_p – specific heat
λ – coefficient of thermal conductivity
f_{wc}, f_{wb} – blow-in parameters
G – the Grashof number
R – the Reynolds number
N – the Nusselt number
N_D – the mass-exchange Nusselt number
P – the Prandtl number
S – the Schmidt number

$$f_{wc} = -\frac{1}{3} R_w \left(\frac{G}{4}\right)^{-1/4}, \quad G = \frac{g\beta_T (T_w - T_\infty) x^3}{\nu^2}, \quad R = \frac{U_\infty x}{\nu}, \quad R_w = \frac{v_w x}{\nu}$$

$$N = \frac{\alpha x}{\lambda}, \quad N_D = \frac{hx}{D}, \quad P = \frac{\nu}{a}, \quad S = \frac{\nu}{D}$$

The subscript w denotes values on the surface, the subscript ∞ denotes values at a great distance from the surface, the subscript 1 indicates the blown-in gas, and the subscript 2 indicates air.

Experimental investigation of mass exchange in a two-component boundary layer at a vertical surface [1] in a forced flow indicates the effect of free convection on the heat and mass release processes.

In the general case the mass diffusion flow of the i-th component in a gas mixture depends on the gradients of the concentration and temperature:

$$j_i = -\rho D \left[\frac{\partial m_i}{\partial y} + a_T \frac{m_i(1 - m_i)}{T} \frac{\partial T}{\partial y}\right]_{i=1,2} \tag{0.1}$$

The first term in (0.1) describes the mass diffusion, the second, the thermal diffusion. The concentrations of the components of the mixture are connected with the density of the mixture as follows:

$$m_i = \rho_i / \rho, \quad m_1 + m_2 = 1$$

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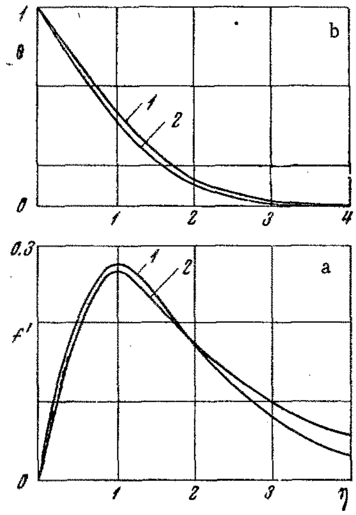

Fig. 1

The heat flow in binary mixtures includes heat transferred by thermal conductivity and diffusion:

$$q = -\lambda\left(\frac{\partial T}{\partial y}\right)_{y=0} + a_T \frac{R_1 M^2 T}{M_1 M_2} j_i \tag{0.2}$$

The second term here defines the diffusion transport of energy (diffusion thermal conductivity).

We consider the convective motion of a binary mixture near a vertical porous plate in a stream with velocity U_∞ and temperature T_∞. Gas of a different nature is blown into the boundary layer through the porous surface of the plate.

When there is joint convection,the one-dimensional heat, mass, and momentum transport are described by the laminar-boundary-layer equations

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty) + g\beta_m(m_1 - m_{1\infty}) \tag{0.3}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{0.4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = -\frac{1}{\rho c_p}\frac{\partial}{\partial y}(q) - \frac{c_{p1} - c_{p2}}{\rho c_p} j_1 \frac{\partial T}{\partial y} \tag{0.5}$$

$$u\frac{\partial m_1}{\partial x} + v\frac{\partial m_1}{\partial y} = -\frac{1}{\rho}\frac{\partial}{\partial y}(j_1) \tag{0.6}$$

with the boundary conditions

$$\begin{gathered} u = 0,\quad v = v_w,\quad T = T_w,\ m_1 = m_{1w}\quad \text{for}\quad y = 0 \\ u = U_\infty,\ T = T_\infty,\ m_1 = m_{1\infty}\quad \text{for}\quad y \to \infty \end{gathered} \tag{0.7}$$

We assume that $T_W > T_\infty$ (for $T_W < T_\infty$ the discussion is no different). The coordinate system is chosen so that the x axis is directed along the surface in the upwards direction while the y axis is perpendicular to it.

In constructing Eqs. (0.3)-(0.6) the physical properties of the medium were taken as constant with the exception of terms expressing the lift, in which the density depends on the temperature and the concentration. In this case we neglect energy dissipation and thermal diffusion, i.e., the second term in (0.1).

1. The Effect of a Forced Flow on Free Convection. As the basic flow we take free convection, and we study the effect on it of a forced flow. To do this we introduce the stream function Ψ, so that

$$u = \frac{\partial \Psi}{\partial y},\qquad v = -\frac{\partial \Psi}{\partial x}$$

satisfy Eqs. (0.4); we also introduce new dependent and independent similarity variables

$$\eta = \frac{c_1 y}{x^{1/4}},\quad \Psi = 4\nu c_1 x^{3/4} f(\eta),\quad c_1 = \left[\frac{g\beta_T(T_w - T_\infty)}{4\nu^2}\right]^{1/4} \tag{1.1}$$

and then instead of (0.3)-(0.6) we obtain the system

$$f'''(\eta) + 3f(\eta)\, f''(\eta) - 2f'^2(\eta) + \theta(\eta) + e\,\varphi(\eta) = 0 \tag{1.2}$$

$$\theta''(\eta) + [3Pf(\eta) + a\varphi'(\eta)]\,\theta'(\eta) + 3cSf(\eta)\ \varphi'(\eta) = 0 \tag{1.3}$$

$$\varphi''(\eta) + 3Sf(\eta)\quad \varphi'(\eta) = 0$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\qquad \varphi(\eta) = \frac{m_1 - m_{1\infty}}{m_{1w} - m_{1\infty}} \tag{1.4}$$

$$a = \frac{c_{p1} - c_{p2}}{c_p}(m_{1w} - m_{1\infty})\frac{P}{S},\qquad c_p = \sum_i c_{pi} m_{iw}$$

$$c = \frac{a_T R_1 M^2 T_w}{c_p M_1 M_2} \frac{(m_{1w} - m_{1\infty})}{(T_w - T_\infty)} \frac{P}{S}$$

$$e = \frac{\beta_m}{\beta_T} \frac{(m_{1w} - m_{1\infty})}{(T_w - T_\infty)}, \qquad \beta_m = \frac{M_2/M_1 - 1}{1 + (M_2/M_1 - 1)\, m_{1w}} \tag{1.5}$$

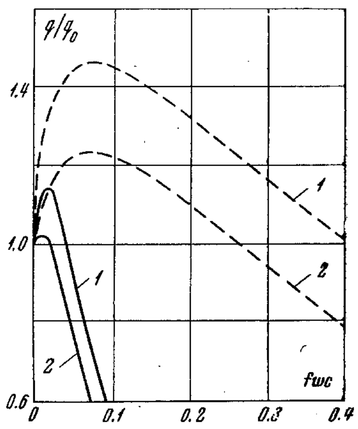

Fig. 2

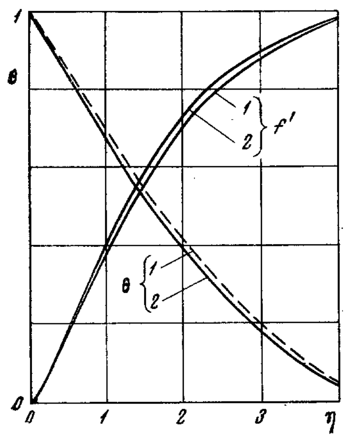

Fig. 3

with the following boundary conditions in the new variables:

$$\begin{aligned} f' &= 0,\ f_w = \mathrm{const},\ \theta = 1,\ \varphi = 1 \quad \text{for} \quad \eta = 0 \\ f' &= {}^1\!/_2 R G^{-1/2},\quad \theta = 0,\quad \varphi = 0 \quad \text{for} \quad \eta \to \infty \end{aligned} \tag{1.6}$$

The non-self-similar boundary condition at infinity for f' does not have a significant effect on the solution. Thus, Fig. 1 gives a comparison of the velocity (a) and temperature (b) profiles for the simplest case of joint convection with P = 0.72 and B = 0.1 when there is no mass exchange or porous input of matter at the wall with the solution of Szewczyk [2] (curve 1) obtained for this case by a different method – expansion in a series in the parameter $R/G^{1/2}$.

The primes in Eqs. (1.2)-(1.4) denote differentiation with respect to η. The parameter $B = R/2G^{1/2}$ defines the effect of the forced flow on free convection. The velocity components u, v are given by the equations

$$u = 4\nu c_1^2 x^{1/2} f'(\eta), \qquad v = \nu c_1 x^{-1/4} [\eta f'(\eta) - 3f(\eta)] \tag{1.7}$$

The boundary condition f_W = const implies that

$$v_w = -3\nu c_1 x^{-1/4} f_w, \quad \text{or} \quad v_w \sim x^{-1/4}.$$

As shown in [3, 4], the behavior of v_W for free and forced convection has a comparatively weak effect on the boundary layer and the heat exchange.

To solve the nonlinear differential equations (1.2)-(1.4) with the boundary conditions (1.6) we use the following iteration process:

1) we choose the zero-order approximation for the functions $f(\eta)$ and $\theta(\eta)$;

2) we substitute the zero-order approximation of $f^{(0)}(\eta)$ in the coefficient of Eq. (1.4) and solve the boundary-value problem. We find the zero-order approximation for $\varphi(\eta)$;

3) substituting the zero-order approximations for the functions $f(\eta)$, $\theta(\eta)$, $\varphi(\eta)$ and their derivatives in Eqs. (1.2), (1.3), we obtain $f^{(1)}(\eta)$, $\theta^{(1)}(\eta)$. Then from (1.4) we find $\varphi^{(1)}(\eta)$, etc.

The above process continues until the difference between two successive approximations for the unknown functions is less than a predefined constant $\varepsilon > 0$ for all η. The linear boundary-value problems ocurring at each iteration are solved by the screw-die method [5]. The quantity a_T is calculated from Eqs. (2-44)-(2-50) of [6] for T_W = 328°K. From computations on an M-220 computer it is possible to calculate the temperature and velocity profiles and the distribution of the component 1 in the boundary layer for joint convection when hydrogen, helium, water vapor, and carbon dioxide are blow in.

The local heat-exchange coefficient is calculated from the equation

$$N = -\left(\frac{G}{4}\right)^{1/4} \left[\theta'(0) + \frac{P}{S} \frac{a_T R_1 M^2 T_w}{c_p M_1 M_2} \frac{(m_{1w} - m_{1\infty})}{(T_w - T_\infty)} \varphi'(0)\right] \tag{1.8}$$

For the mass flow of component 1 (ignoring thermal diffusion), from Eq. (0.1) we find the mass-exchange Nusselt number:

$$N_D = -\left(\frac{G}{4}\right)^{1/4} \varphi'(0) \tag{1.9}$$

Figure 2 gives the ratio of the heat flows when helium is blown in and the temperature conditions are defined by the ratio $T_W/T_\infty = 1.1$. We see that diffusion thermal conductivity has a significant effect on the heat exchange. When small amounts of gas are blown in, the intensity of heat exchange increases; the curves have a maximum. In the case of free convection, when we take account of diffusion thermal conductivity, the maximum occurs when the blow-in parameter is

$$\tfrac{1}{3}R_w\,(\tfrac{1}{4}G)^{-1/4} = 0.015$$

(continuous curve 1), while when we ignore diffusion thermal conductivity, the maximum occurs when the blow-in parameter is

$$\tfrac{1}{3}R_w\,(\tfrac{1}{4}G)^{-1/4} = 0.076$$

(dotted curve).

For a blow-in parameter of 0.05 the difference between the values of q/q_0 calculated with and without diffusion thermal conductivity is 60%. When large volumes of gas are blown in, as a result of which the boundary layer thickens, the heat exchange decreases, and if we take diffusion thermal conductivity into account, q becomes less than q_0 (the value in the absence of blow in). In the case when the diffusion effect is ignored, q becomes equal to q_0 when the blow-in parameter is 0.41.

Curves 2 in Fig. 2 were obtained by taking into account the effect of the forced flow on free convection (B = 2). In this case, when the blow-in parameter is 0.1, the ratio q/q_0 decreases, due to the effect of forced convection, when diffusion thermal conductivity is taken into account, by 14% and, when diffusion thermal conductivity is ignored, by up to 20%.

2. The Effect of Free Convection on Forced Convection. To discuss the effect of free convection on forced convection, we reduce Eqs. (0.3)-(0.6) to ordinary differential equations by introducing the independent variable

$$\eta = y\,(U_\infty / \nu x)^{1/2}$$

and the stream function

$$\Psi = (U_\infty \nu x)^{1/2} f(\eta)$$

In the new variables the velocity components are expressed as follows:

$$u = U_\infty f'(\eta), \quad v = -\frac{1}{2}\left(\frac{U_\infty \nu}{x}\right)^{1/2}[f(\eta) - \eta f'(\eta)] \tag{2.1}$$

and instead of (0.3)-(0.6) we obtain

$$f'''(\eta) + \tfrac{1}{2} f(\eta) f''(\eta) + [\theta(\eta) + e\varphi(\eta)]\frac{G}{R^2} = 0 \tag{2.2}$$

$$\theta''(\eta) + [\tfrac{1}{2} P f(\eta) + a\varphi'(\eta)]\,\theta'(\eta) - \tfrac{1}{2} cS f(\eta)\,\varphi'(\eta) = 0 \tag{2.3}$$

$$\varphi''(\eta) + \tfrac{1}{2} S f(\eta)\,\varphi'(\eta) = 0 \tag{2.4}$$

where $\theta(\eta)$, $\varphi'(\eta)$, a, c, and e are determined by (1.5).

We can make the transformation to ordinary differential equations under the condition that $A = G/R^2 =$ const. This holds exactly if $T_W - T_\infty = Cx^n$, n = const. However, in our case, i.e., when $T_W - T_\infty =$ const, to solve Eqs. (2.2)-(2.4) we use the method of "frozen" coefficients. A test of this method for the simplest model of a joint convection (in the absence of porous input and mass exchange) gave (Fig. 3) satisfactory agreement with Szewczyk's solution [2], in which the parameter G/R^2 is absent from the equation of motion.

The second equation of (2.1) yields an equation for the blow-in (draw-off) parameter:

$$f_{wb} = -\frac{2v_w}{U_\infty}\sqrt{R}$$

The parameter A in the transformed equations (2.2)-(2.4) is independent of η. When this parameter is zero, the equation becomes the equation for forced convection; for large A free convection has an effect on forced flow and heat exchange.

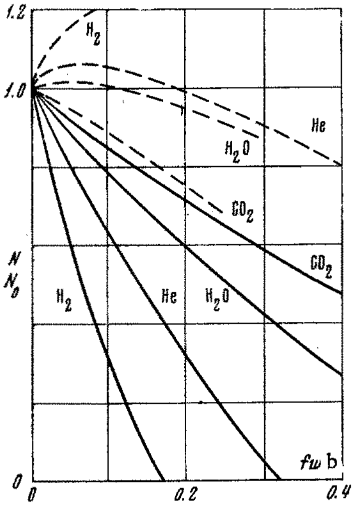

Fig. 4

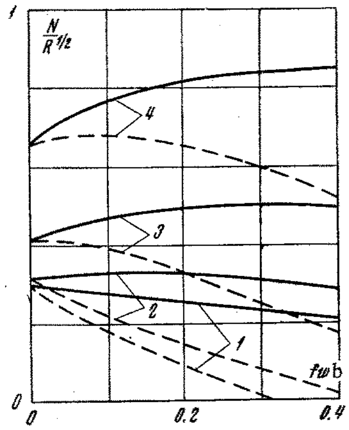

Fig. 5

The boundary conditions (0.7) for the system (2.2)-(2.4) in the new variables for the case of flows which coincide in direction are

$$\begin{array}{l} f' = 0,\ f_w = \text{const},\quad \theta = 1,\ \varphi = 1\quad \text{for}\quad \eta = 0 \\ f' = 1,\ \theta = 0,\ \varphi = 0\ \text{for}\quad \eta \to \infty \end{array} \tag{2.5}$$

when there is joint convection.

When the free and forced convections are in opposite directions, we have $f'(\infty) = -1$ for $\eta \to \infty$ in the boundary conditions (2.5).

Equations (2.2)-(2.4) were solved by the method described in § 1 for free and forced convection in the same and opposite directions.

From the numerical results it is possible to obtain heat and mass flows at a vertical surface when gases of different natures are blown in for various values of G/R^2.

We find the heat flow of a binary mixture from (0.2), from which we obtain

$$N = -R^{1/2}\left[\theta'(0) + \frac{P}{S}\frac{a_T R_1 M^2 T_w}{c_p M_1 M_2}\frac{(m_{1w} - m_{1\infty})}{(T_w - T_\infty)}\varphi'(0)\right] \tag{2.6}$$

The mass-exchange Nusselt number in this case is $N_D = -R^{1/2}\varphi'(0)$.

For a clearer understanding of the physics of the process under consideration we introduce Fig. 4, in which we have constructed curves for N/N_0 as a function of the intensity of the blow in (N_0 is the value of N when there is no mass exchange at the surface) for various gases and for A = 0.10. The graphs show that lighter gases are more effective in lowering the heat exchange than heavy gases in purely forced flow (continous curves). As the effect of free convection increases, the curves (dotted) lie above the continuous ones, i.e., heat exchange increases in proportion to the reduction in the molecular weight of the gas blown in. Due to the effect of free convection the ratio N/N_0 increases by 3% for carbon dioxide and for helium by 70% when the blow-in parameter is 0.1. The effect of diffusion thermal conductivity when helium is blown in and there is joint convection is shown in Fig. 5 for values of A = 0, 0.1, 1, and 10, the dotted curves giving $N/R^{1/2}$ when the diffusion effect is ignored.

When helium is blown in, free convection can be ignored up to $G/R^2 = 0.09$ for $f_{wb} = -0.02$.

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